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An indirect measurement in EEQT

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Abstract. The problem of an indirect measurement is analysed in the so-called event enhanced quantum theory. Two classes of semi-indirect and chain indirect measurements are considered with a view towards applications. A new kind of the EPR experiment is proposed, which can be used for examining the issue of entanglement and that of interacting quantum systems.

1. Introduction

A consistent theory of measurement is still lacking in quantum mechanics (QM). Let us focus on two groups of topics. One question is whether mathematical objects employed by QM have any operational meaning. The other is the problem of a precise description of a large collection of interacting quantum systems that can be regarded as a classical object (classical apparatus is used to probe the quantum system, in general).

These difficulties have been dealt with in many ways. The most popular one is von Neuman's measurement theory [1], which bypasses the basic obstacles by means of brilliant assumptions. In turn, it poses some important questions, e.g. whether the measurement is instantaneous, and if there is an explicit correspondence between measurement operators and the measuring device. The time of a measurement is not specified, and the reasons why a measurement occurs at a particular time instant are unknown. We can ask why one can observe any evolution if the measurement is continuous, as in the quantum Zeno effect or the watchdog effect [2, 3]. There are possible solutions to those problems, such as theories based on a reduction of the wavepacket [4, 5]. Despite these disadvantages, von Neuman's measurement theory works very well and in many cases produces good results [6–8]. Of course, when there are so many serious questions about the theory, there are also some attempts to improve the theory in question. In this respect we find Bohm's quantum potential approach [9, 10], which interprets an interaction with the active role of the enviroment through a specific 'quantum potential'; the many-Hilbert space approach [11–14] which successfully explains difficulties of negative measurement; theories built by quantum opticians to explain quantum jumps, which include such ideas as coherent histories [15, 7], a quantum stochastic calculus [16, 6] and an open systems approach [17, 15, 7].

Following the general theory of open systems [19, 20], the event enhanced quantum theory (EEQT) was proposed and developed in [21–27] to describe a coupled quantum–classical system. The EEQT received an interpretation in terms of a quantum stochastic process [24]. I employ this theory to investigate an issue of an indirect measurement (IM). Within EEQT one does not specify what kind of measurement is involved: direct or indirect.

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I differentiate them in the following way. In the direct measurement, the system consists of two parts: a classical and a quantum one, whereas in the indirect measurement there is one more element—an additional quantum system. This system is supposed to carry a certain amount of information about the former quantum one. In some cases, there is a necessity for taking into account a supplementary quantum system as a part of the measurement device –otherwise treated as a classical apparatus only. In fact, the indirect measurement† has occurred in the study of nonlocal properties of QM [28, 29, 38], in measurements of very weak photon beams (homodyne and heterodyne detection‡ [30–33]) or in observations of very weak classical forces§ [43, 44]. I examine this subject from the EEQT viewpoint.

The IM deserves some attention in experimental and theoretical investigations. It has a number of advantages. It allows us to reduce the influence of the measuring device on the investigated system, to increase sensitivity or precision of the measurement. The performed analysis of IM is easily interpretable and describes an evolution of the measured quantum system. The former property means that a continous measurement is performed that allows us to control, e.g. the detector efficiency [41]. There is also a reason for investigations of the complex measurement on a quantum system. In [45] one can read *'(. . .) In a real experiment the measuring apparatus consists of a series of components. Each component is coupled to the preceding component, and only the first stage in the series directly 'contacts' the system. (. . .) We need only consider the first stage of the measuring apparatus and its interaction with the system (. . .)'*. I would like to point out that in the case of EEQT *'the series of components'* cannot be hidden in the apparatus and may have a great influence on the obtained results.

The paper is organized as follows. Section 2 describes briefly the EEQT, which is extended to give account of two quantum systems, one or two of them being measured. Different kinds of measurement are investigated in section 3. I begin with a short presentation of direct measurement (DM). Descriptions of indirect measurements (a semi-indirect measurement (SIM) and a chain measurement (CIM)) are included in sections 3.2.1 and 3.3. Some nontrivial effects of IM are pointed out in section 3.2.1 (they can be observed even without an interaction between quantum systems). This method could be used for a detection of entanglement of particles. In the last section a classification of measurement methods is provided from the point of view of their possible implementations.

2. Review of EEQT

This presentation is based on the model proposed in [23]‖. For the sake of simplicity we restrict the model to the case of a discrete classical system.

The main idea is to describe the evolution of the coupled classical–quantum system as given by completely positive semigroups. The most natural way of doing this is to use a tensor product of classical and quantum carrier spaces. The evolution is governed by completely positive semigroups rather than by a unitary evolution (the system with dissipation is obtained by coupling classical and quantum system). We assume that the pure states of the quantum system are given by rays in a complex, finite- or infinite-dimensional Hilbert space $\mathcal{H}$. In this paper the Hilbert space $\mathcal{H}$ will be $\mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2$, where $\mathcal{H}_1$ and $\mathcal{H}_2$ are the Hilbert spaces of the first and second quantum system respectively. The observable algebra of the quantum system is the algebra $\mathcal{A}_{\mathrm{q}} = L(\mathcal{H}_1 \otimes \mathcal{H}_2)$ of all bounded operators

† In the case of coupling an additional system.

‡ Homodyne and heterodyne detection can be regarded as a very simple example of a semi-indirect measurement.

§ Gravitational wave detection.

‖ The interested readers may find a more precise description of the theory in [21, 22, 24].

on $\mathcal{H}_1 \otimes \mathcal{H}_2$. The statistical states of the quantum systems are given by positive, weakly continuous functionals w on $\mathcal{A}_{\rm q}$ with $w(I) = 1$. Let $S_{\rm q}$ be a convex set of these states. The elements of $S_{\rm q}$ are positive operators on $\mathcal{H}$ of trace 1.

Let $X_{\rm cl}$ denote the set of pure states of a classical system. As already mentioned the classical system was chosen as a finite one. In this case $X_{\rm cl}$ is finite (with $n+1$ elements). $S_{\rm cl}$ is a set of statistical states of the classical system, which is the space of probability measures on $X_{\rm cl}$. In this case the states $P \in S_{\rm cl}$ are $(n+1)$-tuples $P = (p_0, \ldots, p_n)$, where

$$p_\alpha \geqslant 0 \qquad \text{and} \qquad \sum_\alpha p_\alpha = 1. \tag{2.1}$$

The observable algebra of the classical system $\mathcal{A}_{\rm cl}$ is the Abelian algebra of the complex function on $X_{\rm cl}$, i.e. $\mathcal{A}_{\rm cl} \cong C^{n+1}$†.

The total system has as its algebra:

$$\mathcal{A}_{\rm tot} = \mathcal{A}_{\rm q} \otimes \mathcal{A}_{\rm cl} = L(\mathcal{H}_1 \otimes \mathcal{H}_2) \otimes C^{n+1}. \tag{2.2}$$

It is convenient to realize $\mathcal{A}_{\rm tot}$ as a algebra of operators on some auxiliary Hilbert space:

$$\mathcal{H}_{\rm tot} = \mathcal{H}_1 \otimes \mathcal{H}_2 \otimes C^{n+1}. \tag{2.3}$$

The algebra $\mathcal{A}_{\rm tot}$ is then isomorphic to the algebra of block-diagonal matrices $\mathcal{A} = \operatorname{diag}(a_0, \ldots, a_n)$, whose entries a_α are bounded linear operators on $\mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2$.

There are six special operations on this algebra.

• The embeddings of the quantum and classical algebras into $\mathcal{A}_{\rm tot}$. They read respectively

$$i_{\rm q} : a \in L(\mathcal{H}_1 \otimes \mathcal{H}_2)) \to a \otimes I = \operatorname{diag}_{n+1}(a, \ldots, a) \tag{2.4}$$

$$i_c : \lambda = (\lambda_0, \ldots, \lambda_n) \to \operatorname{diag}(\lambda_0 I, \ldots, \lambda_n I) \tag{2.5}$$

$\lambda_\alpha \in C$.

So, the states of $\mathcal{A}_{\rm tot}$ are represented by block-diagonal matrices

$$\rho = \operatorname{diag}(\rho_0, \ldots, \rho_n) \tag{2.6}$$

$\rho_\alpha \in L(\mathcal{H}_1 \otimes \mathcal{H}_2))$ and $\sum_\alpha \operatorname{Tr}(\rho_\alpha) = 1$.

For the expectation value of an observable $A \in \mathcal{A}_{\rm tot}$ in a state $\Omega \in S_{\rm tot}$ we have $\Omega(A) = \sum_\alpha \operatorname{Tr}(w_\alpha a_\alpha)$. we shall identify states Ω with operators representing them.

• The projectors, which project states of $\mathcal{A}_{\rm tot}$ onto the states of $\mathcal{A}_{\rm q}$, subalgebras of $\mathcal{A}_{\rm q}$ $(L(\mathcal{H}_1), L(\mathcal{H}_2)$ and $\mathcal{A}_{\rm cl}$ are defined as follows:

$$\begin{aligned}
\pi_{\rm q}(\rho) &= \sum_\alpha \rho_\alpha \\
\pi_{q_1}(\rho) &= \sum_\alpha \operatorname{Tr}_{\mathcal{H}_2} \rho_\alpha \\
\pi_{q_2}(\rho) &= \sum_\alpha \operatorname{Tr}_{\mathcal{H}_1} \rho_\alpha \\
\pi_c(\rho) &= (\operatorname{Tr}_{\mathcal{H}_1 \otimes \mathcal{H}_2}(\rho_0), \ldots, \operatorname{Tr}_{\mathcal{H}_1 \otimes \mathcal{H}_2}(\rho_n)).
\end{aligned} \tag{2.7}$$

If we have states: $P = (p_0, \ldots, p_n) \in S_{\rm cl}$ and $\rho_{\rm q} \in S_q$, the state of the joint system may be built in the following way:

$$\rho_{\rm q} \otimes P = \operatorname{diag}(p_0 \rho_{\rm q}, \ldots, p_n \rho_{\rm q}) \tag{2.8}$$

† There is a possibility to build the model where the $X_{\rm cl}$ is an infinite set [26].

where

$$\sum_\alpha p_\alpha = 1$$
$$\mathrm{Tr}_{\mathcal{H}_1\otimes\mathcal{H}_2}\,\rho_{\mathrm{q}} = 1. \tag{2.9}$$

An initial state of the total system can also be built in this fashion. This state represents the situation in which there is no correlation between the states of the classical and quantum systems. The time evolution is given by completely positive (CP) semigroups†.

We deal with a CP semigroup $\alpha^t, t \geqslant 0$ of CP maps of the algebra of observables with the property $\alpha^t(I) = I$. The time evolution of states is given by the one-parameter semigroup of dual maps $\alpha_t : S_{\mathrm{tot}} \to S_{\mathrm{tot}}$ with

$$\alpha_t(\rho)(A) = \rho(\alpha^t(A)). \tag{2.10}$$

In view of theorems by Stinespring and Lindblad [42, 18], any norm continuous semigroup of CP maps α^t must be of the form

$$\alpha^t = \exp(tL) \tag{2.11}$$

with‡

$$L(A) = \mathrm{i}[H, A] + \sum_{i=1}^{N} V_i^* A V_i - \tfrac{1}{2}\left\{\sum_i V_i^* V_i, A\right\} \tag{2.12}$$

where

$$\sum_i V_i^* V_i \in \mathcal{A}_{\mathrm{tot}} \tag{2.13}$$

and

$$V_i^* A V_i \in \mathcal{A}_{\mathrm{tot}} \qquad \text{whenever } A \in \mathcal{A}_{\mathrm{tot}}. \tag{2.14}$$

V_i will be called coupling operators, because they define the coupling between classical and quantum system§. H is an arbitrary Hermitan operator in $\mathcal{A}_{\mathrm{tot}} : H = H^* \in \mathcal{A}_{\mathrm{tot}}$. Let us denote $\rho(t) = \alpha_t(\rho)$, so the evolution (2.12) of observables of the system leads to the Liouville evolution equation for states:

$$\dot\rho(t) = -\mathrm{i}[H, \rho(t)] + \sum_i V_i \rho(t) V_i^* - \tfrac{1}{2}\left\{\sum_i V_i^* V_i, \rho(t)\right\}. \tag{2.15}$$

3. Quantum measurement

3.1. The direct measurement

The DM in EEQT is described in terms of:
the classical space is C^{n+1},
the quantum space is a Hilbert space $\mathcal{H}$,
the total space is a tensor product of these spaces $\mathcal{H} \otimes C^{n+1}$.

The elements of the quantum algebra $L(\mathcal{H})$ are linear bounded operators on a Hilbers space $\mathcal{H}$. Elements of the classical algebra are probabilistic measures on a set of classical events. The state of the total (calssical–quantum) system is described:

$$\rho(t) = \mathrm{diag}\{\rho_0(t), \ldots, \rho_n(t)\} \tag{3.1}$$

† A brief discussion on why this mathematical structure is used here can be found in [22, 42, 18, 19].
‡ Brackets [,], { , } denote commutator and anticommutator respectively.
§ It is important to observe that the operators V_i do not need to belong to the $\mathcal{A}_{\mathrm{tot}}$.

where

$$\rho_i(t) = p_i(t)\rho_{\mathrm{q}}(t). \tag{3.2}$$

In this case we have two projectors:

- onto the classical space

$$p_i(t) = \mathrm{Tr}\,\rho_i(t) \tag{3.3}$$

- onto the quantum space

$$\rho_{\mathrm{q}}(t) = \sum_i \rho_i. \tag{3.4}$$

The evolution of the total system is described by equation (2.15). Using the projections (3.3) and (3.4), we get an evolution of the quantum and classical systems respectively

$$\dot{p}_i(t) = \frac{\mathrm{d}}{\mathrm{d}t}\,\mathrm{Tr}\,\rho_i(t) = \mathrm{Tr}\,\dot{\rho}_i(t). \tag{3.5}$$

It is important to make sure this operation is allowed†. We will assume that equation (3.5) holds true.

$$\dot{\rho}_{\mathrm{q}}(t) = \frac{\mathrm{d}}{\mathrm{d}t}\sum_i \rho_i(t) = \sum_i \dot{\rho}_i(t). \tag{3.6}$$

Because i runs over the finite set there is no question about the equation (3.6), but in other cases problems may arise. Moreover we assume the following shape of the operators H and V, which are present in the equation (2.15):

$$H = \mathrm{diag}\{H_0, \ldots, H_n\} \tag{3.7}$$

and

$$H_i = \hat{H} \tag{3.8}$$

while V_i satisfy the general assumption (2.13), (2.14). Using notations (3.1), (3.7), (3.8), equation (3.15) takes the form:

$$\dot{\rho}_i(t) = -\mathrm{i}[\hat{H}, \rho_i(t)] + \sum_{kj} V_{ij}^k \rho_j(t) V_{ji}^{k*} - \tfrac{1}{2}\left\{\sum_{kj} V_{ij}^{k*} V_{ji}^k, \rho_i(t)\right\} \tag{3.9}$$

where V^k are $(n+1)\times(n+1)$ matrices, whose entries are linear bounded operators in $L(\mathcal{H})$. Then

$$\dot{\rho}_i(t) = -\mathrm{i}[\hat{H}, \rho_i(t)] + \sum_{kj} V_{ij}^k \rho_j(t) V_{ij}^{*k} - \tfrac{1}{2}\left\{\sum_{kj} V_{ji}^{*k} V_{ji}^k, \rho_i(t)\right\}. \tag{3.10}$$

Inserting (3.10) into (3.5) and (3.6) we get

$$\dot{p}_i(t) = \mathrm{Tr}\left(-\mathrm{i}[\hat{H}, p_i(t)\rho_{\mathrm{q}}(t)] + \sum_{kj} V_{ij}^k p_i(t)\rho_{\mathrm{q}}(t) V_{ij}^{*k} - \tfrac{1}{2}\left\{\sum_{kj} V_{ji}^{*k} V_{ji}^k, p_i(t)\rho_{\mathrm{q}}(t)\right\}\right) \tag{3.11}$$

$$\dot{p}_i(t) = \sum_{kj} p_j(t)\,\mathrm{Tr}(V_{ij}^{*k} V_{ij}^k \rho_{\mathrm{q}}(t)) - p_i(t)\sum_{kj} \mathrm{Tr}(V_{ji}^{*k} V_{ji}^k \rho_{\mathrm{q}}(t)) \tag{3.12}$$

† We assume that basis vector of Hilbert space do not depend on time and a derivative of an operator is also a trace class operator.

and

$$\rho_{\rm q}(t) = \sum_i \left(-\,{\rm i}[\hat{H}, p_i(t)\rho_{\rm q}(t)] + \sum_{kj} V^k_{ij} p_j(t)\rho_{\rm q}(t) V^{*k}_{ij} - \tfrac{1}{2}\left\{ \sum_{kj} V^{*k}_{ji} V^k_{ji}, p_i(t)\rho_{\rm q}(t)\right\}\right) \tag{3.13}$$

$$\rho_{\rm q}(t) = -{\rm i}[\hat{H}, \rho_{\rm q}(t)] + \sum_{ijk} p_j(t) V^k_{ij}\rho_{\rm q}(t) V^{*k}_{ij} - \tfrac{1}{2}\sum_{ikj} p_i(t)\left\{V^{*k}_{ji} V^k_{ji}, \rho_{\rm q}(t)\right\}. \tag{3.14}$$

Equations (3.12) and (3.14) give us the evolution of the classical and quantum subsystems. The choice of the operators V^k is very important, because we can obtain a completely different evolution simply by rearanging entries of V^k (see [41]).

3.2. The indirect measurement

In this section I consider two kinds of IM. The first is a SIM. This is the case in which we couple a quantum system to the investigated one and perform a measurement over the total coupled quantum system† (pair of coupled ones). The second is a CIM. In CIM the measurement is performed separately over a quantum system, which in turn is a member of a coupled pair‡.

3.2.1. The semi-indirect measurement. We shall consider the following system.

- There are two quantum systems. They are described within Hilbert spaces $\mathcal{H}_1$ and $\mathcal{H}_2$ and governed by Hamiltonians $\hat{H}_1$ and $\hat{H}_2$ respectively. Moreover we assume that the interaction between quantum systems is described by a Hamiltonian $\tilde{H}_{\rm int}$ given in the form $\tilde{H}_{\rm int} = \bar{H}_1 \otimes \bar{H}_2$ and acting on a tensor product of $\mathcal{H}_1 \otimes \mathcal{H}_2$.
- The classical system, which consists of a discrete set, describes measurement events. This classical system will be considered as an $(n+1)$-dimensional probability space, where n is the number of distinguishable classical states§.

The most important issue is that the measurement is done over the whole (coupled) quantum system. Following the above assumption let us define the system: The Hilbert space of the total system is

$$\mathcal{H}_{\rm tot} = \mathcal{H}_1 \otimes \mathcal{H}_2 \otimes C^{n+1}. \tag{3.15}$$

The Hamiltonian of the coupled quantum system is

$$\tilde{H} = \tilde{H}_1 + \tilde{H}_2 + \tilde{H}_{\rm int} \tag{3.16}$$

where

$$\begin{aligned} \tilde{H}_1 &= \hat{H}_1 \otimes I \\ \tilde{H}_2 &= I \otimes \hat{H}_2 \\ \tilde{H}_{\rm int} &= \bar{H}_1 \otimes \bar{H}_2. \end{aligned} \tag{3.17}$$

I is an identity operator, $\hat{H}_1, \hat{H}_2$ are the Hamiltonian operators of the first and second quantum system and $\bar{H}_1 \otimes \bar{H}_2$ are the operators describing the mutual interaction of quantum systems. The Hamiltonian of the total system is in the form

$$H = {\rm diag}_{n+1}(\tilde{H}, \ldots, \tilde{H}). \tag{3.18}$$

† The simplest example of such a measurement is a homodyne detection.
‡ Very simple example of the CIM is a measurement done on EPR particles.
§ Distinguishable by a classical device.

The state of the total system has the form

$$\rho(t) = \mathrm{diag}(\rho_0(t), \ldots, \rho_n(t)) \tag{3.19}$$
$$\rho_i(t) = p_i(t)\rho^{(1)}(t) \otimes \rho^{(2)}(t). \tag{3.20}$$

The shape of $\rho(t)$ is the result of the assumption about the total space. In this case the states are block-diagonal matrices $(n+1)\times(n+1)$ whose diagonal is composed of operators belonging to $L(\mathcal{H}_1 \otimes \mathcal{H}_2)$. For the sake of simplicity we denote the projections (2.7) as follows:

- onto the classical space

$$p_i(t) = \mathrm{Tr}_{\mathcal{H}_1} \mathrm{Tr}_{\mathcal{H}_2}\, \rho \tag{3.21}$$

- onto the Hilbert spaces $\mathcal{H}_1$ and $\mathcal{H}_2$ respectively

$$\rho^{(1)}(t) = \mathrm{Tr}_{\mathcal{H}_2} \sum_i \rho_i(t) \tag{3.22}$$
$$\rho^{(2)}(t) = \mathrm{Tr}_{\mathcal{H}_1} \sum_i \rho_i(t). \tag{3.23}$$

The evolution of the total system is governed by the equation (2.15). In the case of a 'total' measurement we assume the following shape of the dissipation operators V_i, which have to satisfy (2.13), (2.14).

$$V_i = (P_1 \otimes P_2)^i_{kl}. \tag{3.24}$$

Rewriting the equation (2.15) in the form of matrix elements (3.19) and using assumptions (3.16), (3.24) we obtain

$$\begin{aligned}\dot{\rho}_i = {} & -\mathrm{i}[\tilde{H}_1, \rho_i(t)] - \mathrm{i}[\tilde{H}_2, \rho_i(t)] - \mathrm{i}[\tilde{H}_{\mathrm{int}}, \rho_i(t)] + \sum_{k,l}((P_1 \otimes P_2)^k_{il}\rho_l(t)(P_1^* \otimes P_2^*)^k_{li} \\ & -\tfrac{1}{2}\{(P_1^* \otimes P_2^*)^k_{il}(P_1 \otimes P_2)^k_{li}, \rho_i(t)\}).\end{aligned} \tag{3.25}$$

Substituting 3.17, 3.19 into 3.25 we obtain

$$\begin{aligned}\dot{\rho}_i(t) = {} & -\mathrm{i}[\hat{H}_1 \otimes I, p_i(t)\rho^{(1)}(t) \otimes \rho^{(2)}(t)] - \mathrm{i}[I \otimes \hat{H}_2, p_i(t)\rho^{(1)}(t) \otimes \rho^{(2)}(t)] \\ & -\mathrm{i}[\bar{H}_1 \otimes \bar{H}_2, p_i(t)\rho^{(1)}(t) \otimes \rho^{(2)}(t)] \\ & + \sum_{k,l}((P_1)^k_{il} \otimes (P_2)^k_{il})p_l(t)\rho^{(1)}(t) \otimes \rho^{(2)}(t)((P_1^*)^k_{li} \otimes (P_2^*)^k_{li}) \\ & -\tfrac{1}{2}\left\{\sum_{k,l}((P_1^*)^k_{li}(P_1)^k_{li} \otimes (P_2^*)^k_{li}(P_2)^k_{li}), p_i(t)\rho^{(1)}(t) \otimes \rho^{(2)}(t)\right\}.\end{aligned} \tag{3.26}$$

In the case of such a system three features are worth further consideration:

- an evolution of a classical state (simply stated, we are interested in outcomes which are produced by such a system),
- an evolution of the first quantum system (in other words, what kind of change is introduced by the measurement into the investigated system),
- what kind of properties of a quantum system can be measured by this method.

Classical system. In order to answer the above questions we use the projections (3.21)–(3.23). Let us see what kind of results can be obtained:

$$\frac{\mathrm{d}}{\mathrm{d}t}\dot{p}_i(t) = \frac{\mathrm{d}}{\mathrm{d}t}(\mathrm{Tr}_{\mathcal{H}_1} \mathrm{Tr}_{\mathcal{H}_2}\, \rho_i(t)) = \mathrm{Tr}_{\mathcal{H}_1} \mathrm{Tr}_{\mathcal{H}_2} \frac{\mathrm{d}}{\mathrm{d}t}\rho_i(t). \tag{3.27}$$

Applying the operation (3.27) to equation (3.26), using the properties of the trace operation and (2.1) we obtain:

$$\dot{p}_i(t) = \sum_{kl} p_l(t)\,\mathrm{Tr}_{\mathcal{H}_1}(P^{*k}_{1il}P^{k}_{1il}\rho^{(1)}(t))\,\mathrm{Tr}_{\mathcal{H}_2}(P^{*k}_{2il}P^{k}_{2il}\rho^{(2)}(t))$$
$$-p_i(t)\sum_{kl}\mathrm{Tr}_{\mathcal{H}_1}(P^{*k}_{1li}P^{k}_{1li}\rho^{(1)}(t))\,\mathrm{Tr}_{\mathcal{H}_2}(P^{*k}_{2li}P^{k}_{2li}\rho^{(2)}(t)). \tag{3.28}$$

For the sake of simplicity we will not differentiate operators P by writing the index of a Hilbert space on which they act. We assume that they act on a space to which ρ belongs too. Then

$$\dot{p}_i(t) = \sum_{kl} p_l(t)\,\mathrm{Tr}_{\mathcal{H}_1}(P^{*k}_{il}P^{k}_{il}\rho^{(1)}(t))\,\mathrm{Tr}_{\mathcal{H}_2}(P^{*k}_{il}P^{k}_{il}\rho^{(2)}(t))$$
$$-p_i(t)\sum_{kl}\mathrm{Tr}_{\mathcal{H}_1}(P^{*k}_{li}P^{k}_{li}\rho^{(1)}(t))\,\mathrm{Tr}_{\mathcal{H}_2}(P^{*k}_{li}P^{k}_{li}\rho^{(2)}(t)). \tag{3.29}$$

Investigated quantum system. Using (3.22) we get

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho^{(1)}(t) = \frac{\mathrm{d}}{\mathrm{d}t}\left(\sum_i \mathrm{Tr}_{\mathcal{H}_2}\,\rho_i(t)\right) = \sum_i \mathrm{Tr}_{\mathcal{H}_2}\frac{\mathrm{d}}{\mathrm{d}t}\rho_i(t). \tag{3.30}$$

Under similar conditions as in (3.27), substituting (3.26) into (3.30), using (2.1) and the trace properties we get

$$\dot{\rho}^{(1)}(t) = -\mathrm{i}[\hat{H}_1,\rho^{(1)}(t)](\mathrm{Tr}_{\mathcal{H}_2}\,\rho^{(2)}(t)) - \mathrm{i}[\bar{H}_1,\rho^{(1)}(t)](\mathrm{Tr}_{\mathcal{H}_2}\,\bar{H}_2\rho^{(2)}(t))$$
$$+\sum_{ikl} p_l(t)(\mathrm{Tr}_{\mathcal{H}_2}\,P^{*k}_{il}P^{k}_{il}\rho^{(2)}(t))P^{k}_{il}\rho^{(1)}(t)P^{*k}_{il}$$
$$-\tfrac{1}{2}\sum_{ikl} p_i(t)(\mathrm{Tr}_{\mathcal{H}_2}\,P^{*k}_{li}P^{k}_{li}\rho^{(2)}(t))\{P^{*k}_{li}P^{k}_{li},\rho^{(1)}(t)\}. \tag{3.31}$$

In the special case when

$$\tilde{H}_{\mathrm{int}} = 0 \tag{3.32}$$

it becomes

$$\dot{\rho}^{(1)}(t) = -\mathrm{i}[\hat{H}_1,\rho^{(1)}(t)](\mathrm{Tr}_{\mathcal{H}_2}\,\rho^{(2)}(t)) + \sum_{ikl} p_l(t)(\mathrm{Tr}_{\mathcal{H}_2}\,P^{*k}_{il}P^{k}_{il}\rho^{(2)}(t))P^{k}_{il}\rho^{(1)}(t)P^{*k}_{il}$$
$$-\tfrac{1}{2}\sum_{ikl} p_i(t)(\mathrm{Tr}_{\mathcal{H}_2}\,P^{*k}_{li}P^{k}_{li}\rho^{(2)}(t))\{P^{*k}_{li}P^{k}_{li},\rho^{(1)}(t)\}. \tag{3.33}$$

It is seen that the only difference between the measurement over interacting and noninteracting quantum systems consist in the part connected with the intraction of quantum systems. But despite (3.32), the second quantum system still has a significant influence on the measured system. (The term depending on the state of the second system is present in the dissipative part of equation (3.33).)

The additional system. From the projection operation (3.23) we see

$$\dot{\rho}^{(2)}(t) = \frac{\mathrm{d}}{\mathrm{d}t}\left(\sum_i \mathrm{Tr}_{\mathcal{H}_1}\right) = \sum_i \mathrm{Tr}_{\mathcal{H}_1}\,\dot{\rho}_i(t). \tag{3.34}$$

Under similar assumptions as in equation (3.27), using equations (3.26), (2.1) and the properties of the trace operation we get

$$\begin{aligned}\dot{\rho}^{(2)}(t) = &-\mathrm{i}[\hat{H}_2, \rho^{(2)}(t)](\mathrm{Tr}_{\mathcal{H}_1}\, \rho^{(1)}(t)) - \mathrm{i}[\bar{H}_2, \rho^{(2)}(t)](\mathrm{Tr}_{\mathcal{H}_1}\, \bar{H}_1\rho^{(1)}(t)) \\ &+ \sum_{ikl} p_l(t)(\mathrm{Tr}_{\mathcal{H}_1}\, P^{*k}_{il}P^k_{il}\rho^{(1)}(t))P^k_{il}\rho^{(2)}(t)P^{*k}_{il} \\ &- \tfrac{1}{2}\sum_{ikl} p_i(t)(\mathrm{Tr}_{\mathcal{H}_1}\, P^{*k}_{li}P^k_{li}\rho^{(1)}(t))\{P^{*k}_{li}P^k_{li}, \rho^{(2)}(t)\}.\end{aligned} \tag{3.35}$$

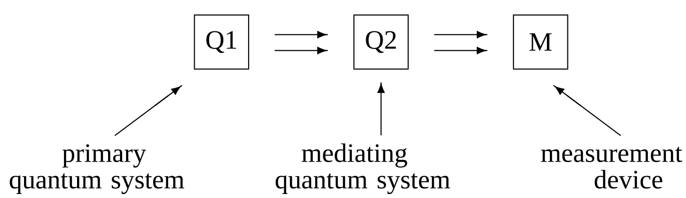
It is very interesting that even in the case of the SIM the result obtained in this way is not just a simple combination of those for two systems investigated separately.

3.3. The chain indirect measurement

We are considering the system which consists of:

- two quantum systems (measured and mediating),
- one classical system (measurement device).

The basic assumptions are: (a) there is an interaction between quantum systems, (b) the measurement is performed over the mediating quantum system. The flow of information can be seen in the illustration:



According to section 2 this system† is described within the Hilbert space:

$$\mathcal{H}_{\mathrm{tot}} = \mathcal{H}_1 \otimes \mathcal{H}_2 \otimes \mathcal{C}^{n+1} \tag{3.36}$$

where $\mathcal{H}_1, \mathcal{H}_2$ are the respective Hilbert spaces of the quantum systems and $\mathcal{C}^{n+1}$ the space of classical events (it is the $(n+1)$-dimensional probability space). The state of the total system is represented by

$$\rho(t) = p(t)\rho^{(1)}(t) \otimes \rho^{(2)}(t) \tag{3.37}$$

where $p(t)$ is $(n+1)$-tuple $p(t) = \{p_0(t), \ldots, p_n(t)\}$. $\rho^{(1)}(t), \rho^{(2)}(t)$ describe the states of quantum systems. We assume also the total Hamiltonian

$$\tilde{H} = \tilde{H}_1 + \tilde{H}_2 + \tilde{H}_{\mathrm{int}} \tag{3.38}$$

where $\tilde{H}_1, \tilde{H}_2$ are Hamiltonians of the first and the second system respectively. $\tilde{H}_{\mathrm{int}}$ is a Hamiltonian describing an interaction between quantum systems. In order to define the evolution of an open system, we have to define dissipative operators. They must satisfy the general conditions: (2.13), (2.14) and we take them in the form

$$\{V_i\}_{kl} = I \otimes P^i_{kl}. \tag{3.39}$$

This is because of the assumption that the measurement is performed over the mediating quantum system. $\{V_i\}_{kl}$ are bounded linear operators on $\mathcal{H}_1 \otimes \mathcal{H}_2$ ($\{V_i\}_{kl} \in L(\mathcal{H}_1 \otimes \mathcal{H}_2)$) and $P_{kl} \in L(\mathcal{H}_2)$. In the present case the same projections as in section 3.2.1 ((3.21), (3.22),

† As well as in section 3.2.1.

(3.23)) arise. Within this mathematical framework we can give a more precise description of the Hamiltonian operators

$$\begin{aligned}\tilde{H}_1 &= \hat{H}_1 \otimes I \\ \tilde{H}_2 &= I \otimes \hat{H}_2 \\ \tilde{H}_{\text{int}} &= \bar{H}_1 \otimes \bar{H}_2.\end{aligned} \tag{3.40}$$

So the equation (2.15) takes the general form:

$$\begin{aligned}\dot{\rho}_i(t) = -\mathrm{i}[\tilde{H}_1, \rho_i(t)] - \mathrm{i}[\tilde{H}_2, \rho_i(t)] - \mathrm{i}[\tilde{H}_{\text{int}}, \rho_i(t)] \\ + \sum_{kl} V_{il}^k \rho_l(t) V_{li}^{k*} - \tfrac{1}{2}\left\{\sum_{kl} V_{il}^{k*} V_{li}^k, \rho_i(t)\right\}.\end{aligned} \tag{3.41}$$

To be specialized to concrete situations, see below.

The classical system. Using the decomposition (3.20), assumption (2.1) and the trace properties we get the classical state evolution (cf also equation (3.27))

$$\dot{p}_i(t) = \sum_{kl} p_l(t)\,\mathrm{Tr}_{\mathcal{H}_2}(P_{il}^{*k} P_{il}^k \rho^{(2)}(t)) - p_i(t) \sum_{kl} \mathrm{Tr}_{\mathcal{H}_2}(P_{li}^{*k} P_{li} \rho^{(2)}(t)). \tag{3.42}$$

The measured system. The evolution of the measured quantum system can be obtained by using (3.41), (3.22), (3.38), (3.40) and the trace properties.

$$\dot{\rho}^{(1)}(t) = -\mathrm{i}[\hat{H}_1, \rho^{(1)}(t)]\,\mathrm{Tr}_{\mathcal{H}_2}(\rho^{(2)}(t)) \tag{3.43}$$

$$- \mathrm{i}[\bar{H}_1, \rho^{(1)}(t)]\,\mathrm{Tr}_{\mathcal{H}_2}(\bar{H}_2 \rho^{(2)}(t)) \tag{3.44}$$

$$+ \sum_{ikl} p_l(t)\,\mathrm{Tr}_{\mathcal{H}_2}(P_{il}^{*k} P_{il}^k \rho^{(2)}(t))\rho^{(1)}(t) \tag{3.45}$$

$$- \sum_{ikl} p_i(t)\,\mathrm{Tr}_{\mathcal{H}_2}(P_{li}^{*k} P_{li}^k \rho^{(2)}(t))\rho^{(1)}(t). \tag{3.46}$$

Applying property (2.1) we see that

$$\sum_{ikl} p_l(t)(\mathrm{Tr}_{\mathcal{H}_2} P_{il}^{*k}) P_{il}^k)\rho^{(2)}(t)) - \sum_{ikl} p_i(t)\,\mathrm{Tr}_{\mathcal{H}_2}(P_{li}^{*k} P_{li}^k)\rho^{(2)}(t)) = 0 \tag{3.47}$$

so finally equation (3.46) takes the form

$$\dot{\rho}^{(1)}(t) = -\mathrm{i}[\hat{H}_1, \rho^{(1)}(t)]\,\mathrm{Tr}_{\mathcal{H}_2}(\rho^{(2)}(t)) - \mathrm{i}[\bar{H}_1, \rho^{(1)}(t)]\,\mathrm{Tr}_{\mathcal{H}_2}(\bar{H}_2 \rho^{(2)}(t)). \tag{3.48}$$

Introducing a new effective Hamiltonian, which depends on time:

$$H_{\text{new}}(t) = (\mathrm{Tr}_{\mathcal{H}_2} \rho^{(2)}(t))\hat{H}_1 + (\mathrm{Tr}_{\mathcal{H}_2} \bar{H}_2 \rho^{(2)}(t))\bar{H}_1 \tag{3.49}$$

equation (3.48) may be rewritten:

$$\dot{\rho}^{(1)}(t) = -\mathrm{i}[H_{\text{new}}(t), \rho^{(1)}(t)]. \tag{3.50}$$

We see here that the evolution of the primary system is of a Hamiltonian type.

The mediating system. Applying the decomposition (3.20) and the projection (3.23) to equation (3.41) we get

$$\begin{aligned}\dot{\rho}^{(2)}(t) = -\mathrm{i}[\hat{H}_2, \rho^{(2)}(t)]\,\mathrm{Tr}_{\mathcal{H}_1}\,\rho^{(1)}(t) - \mathrm{i}[\bar{H}_2, \rho^{(2)}(t)]\,\mathrm{Tr}_{\mathcal{H}_1}(\bar{H}_1\rho^{(1)}(t))\\ + \sum_{ikl} p_l(t)(P^k_{il}\rho^{(2)}(t)P^{*k}_{il})\,\mathrm{Tr}_{\mathcal{H}_1}\,\rho^{(1)}(t)\\ -\tfrac{1}{2}\sum_{ikl} p_i(t)\{P^{*k}_{li}P_{li}, \rho^{(2)}(t)\}\,\mathrm{Tr}_{\mathcal{H}_1}\,\rho^{(1)}(t).\end{aligned} \tag{3.51}$$

Equation (3.51) cannot be simplified in the same way as (3.46) because of the anticommutator in the fourth entry.

3.4. Comparison of CIM and SIM

3.4.1. The classical results. The difference between SIM and CIM is such that the state of $\rho^{(1)}(t)$ has an explicite influence on the result of the measurement. In CIM this influence is strictly indirect. In both IMs a special treatment of the system is required in order to obtain information about the investigated quantum system. The case of CIM is more complicated then SIM because a special choice of measurement operators is required, which should be sensitive to the presence of the investigated system. Moreover in both IMs we have to unravel information about $\rho^{(1)}$ from the 'total' output (e.g. see [30, 31]). The most interesting fact is that the nonlocal properties of QM (without the interaction of quantum systems) can be observed only in SIM, whereas in CIM we observe the mediating quantum system only.

3.4.2. The primary quantum system. When we compare equations (3.33) (SIM) and (3.50) (CIM) we see that both kinds of measurement generate a different evolution of a system. Equation (3.33) gives account of a typical dissipative evolution, similar to the direct measurement, whereas in CIM the evolution of the primary system is that of a Hamiltonian type. Of course, the new Hamiltonian depends on time, which may give us a complicated task to solve this kind of equation. But the most important fact is that CIM offers a possibility to reduce the influence of a measurement by allowing for a special treatment of a quantum state, for instance by using a feedback (similar to [39, 40]). It has also the great advantage of generating well-defined states suitable for encoding classical information. This property can be used in the quantum transfer of classical information [41].

3.4.3. The mediating and additional quantum state. The equations describing the evolution of $\rho^{(2)}$ also expresses the difference between IM and DM. In (3.51) (CIM) we can find out what kind of properties the operators P^k_{ij} should possess to be able to measure the state of $\rho^{(1)}$. Furthermore the properties of $\rho^{(1)}$ can be measured (or well seen) only when the system described by $\rho^{(2)}$ has been chosen appropriately. (The meaning of the word 'appropriately' depends on the particular choice of the system.) The main difference, which is seen in (3.35), (3.51), lies in the influence of the primary system on the second one. Roughly speaking, in CIM the mediating state evolution is multiplied by the trace of $\rho^{(1)}$ whereas in SIM this dependence is much more complicated. Moreover in SIM the influence is present even when there is no coupling ($\tilde{H}_{\mathrm{int}} = 0$) between quantum systems. The measurement induces a mutual influence of the quantum systems.

4. Conclusions

The general advantages of applying EEQT as a measurement theory are:

- it satisfies all demands of a measurement theory [23],
- it allows us to define a broad class of detectors following the same operators acting on a Hilbert space [41],
- it describes a continuous measurement process.

4.1. CIM

This method of measurement should be applied in two cases. First, when it is very important to reduce the influence of a measurement device on the investigated system (and when it is possible to obtain the required information). The influence is even weaker than in DM†. Second, when we want to generate a particular state in order to encode some information. This last ability may be very important for quantum transmission of classical information.

We stress once again that not every piece of information is accessible this way. Obtaining information about the quantum state $\rho^{(1)}$ requires suitable measurement operators. They have to be sensitive to the changes in the evolution of $\rho^{(2)}$ caused by $\rho^{(1)}$. CIM creates a new possibility for a measurement but requires a special treatment of the system. In order to obtain any results in CIM two issues are extremely important:

- a proper choice of the mediating system (it has to be sensitive to the changes caused by the primary system),
- the choice of the dissipative operators (the proper coupling of the quantum systems with the classical one—if this is done incorrectly, no information will be received).

4.2. SIM

There are some advantages in comparison with CIM and DM.

- The main one is the possibility of a measurement of very weak properties (hardly observable) of a system. This cannot be measured by DM. (This is done through the modulation of the base signal by a weak signal. When combined with the special treatment of the output, it can give us very interesting results.) We may see this kind of measurement in a homodyne measurement. If the local oscillator is chosen properly, it allows us to observe the features of the system which cannot be seen in any other way.
- The SIM gives also a possibility for measurement of any observable, whereas CIM requires observables that are sensitive to special properties of the system.

5. The possible usage

Comparing the advantages and disadvantages of the three methods of quantum measurement described, DM, SIM and CIM, we may figure where each of those methods may be more profitable. DM is the simplest measurement‡ so it should be used if the measurement can be done in this way, because the outcomes do not include any impact of other systems. The only problem which can appear at this moment is the preparation of the system, because usually it is very difficult to reduce an influence of other systems and receive a pure coupling of a quantum and classical device. From this point of view, SIM should commonly be used in the situation when the removal of an influence of other systems is impossible. SIM

† See equation (3.50).

‡ But only from the theoretical point of view. In the case of an experiment it may be the most difficult one.

offers also a possibility to modulate a base signal by another quantum system in order to get information about the second system. The last procedure (CIM) described in this paper may be applied in two situations: to minimize the influence of the measurement device on the system or to generate some kind of quantum state.

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